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Superscaling in inclusive electron-nucleus scattering

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Abstract

We investigate the degree to which the scaling functions $F(\psi')$ derived from cross sections for inclusive electron-nucleus quasi-elastic scattering define the *same* function for *different* nuclei. In the region where the scaling variable $\psi' < 0$, we find that this superscaling is experimentally realized to a high degree.

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I. INTRODUCTION

The use of scaling and the application of dimensional analysis to cross sections for inclusive scattering of a weakly interacting probe from the constituents of a composite system have been important tools for the development of new insights in physics. Examples for such processes are the scattering of keV electrons from electrons bound in atoms [1], the scattering of eV neutrons from atoms in solids or liquids [2], deep inelastic scattering of GeV energy leptons from the quarks in the nucleon [3] and, of particular interest for the present letter, quasi-elastic scattering of electrons in the energy range of 100's of MeV to several GeV from nucleons in nuclei [4]. Despite the extraordinary range of energy and momentum transfer for which scaling has been studied, the conceptual basis for describing this phenomenon has many features in common.

The inclusive cross sections for these processes in general depend explicitly on two independent variables – the energy ω and momentum $\vec{q}$ transferred by the probe to the constituent. *Scaling* means that, in the asymptotic regime of large q and ω, the cross sections depend on a *single* variable $z = z(q, \omega)$, itself a function of ω and q. This property results essentially from the kinematics of the scattering process of the probe by the moving constituent, which as a consequence of the recoil momentum received, is ejected essentially quasi-freely from the composite system.

The interest in scaling phenomena originates from two distinct sources:

- The observation of the occurrence (or nonoccurrence) of scaling yields information on the domination (or not) of the quasi-free scattering process or the contribution of other reaction mechanisms (which in general do not scale). This provides *experimental* insight into the reaction mechanism, knowledge which is a prerequisite for a quantitative understanding of the cross section.

- The function to which the data scale is closely related to the momentum distribution of the constituents in the composite target. This provides interesting experimental knowledge on the dynamics of the bound system.

For quasi-elastic electron-nucleus scattering, data are available over a large range of (q, ω) and for several nuclei. These data have been found to scale in a major part of the kinematical region studied. When analyzed in terms of the scaling variable y [4], which is close to the component $k_{//}$ of a bound nucleon's momentum parallel to $\vec{q}$, the data exhibit scaling for $y < 0$ (roughly $\omega < |Q^2|/2m_N$, with m_N the nucleon mass, where $Q^2 = \omega^2 - q^2 < 0$), that is, the region in which quasi-elastic scattering dominates. Much of the past work has concentrated on the study of the scaling properties of the response in the low-ω tail of the quasi-elastic peak where y is large and negative. Detailed quantitative studies of the conditions under which scaling occurs and the impact of adverse effects such as final-state interactions [5], the spread of the spectral function $S(k, E)$ in removal energy E, and the contributions of other reaction mechanisms have been made. For a review see [6].

Past applications of scaling focused on individual nuclei and nuclear matter using the data for different kinematics in the range $q = 0.5 - 2$ GeV/c and $\omega = 0.1 - 3$ GeV. In the present letter, we explore a novel aspect: rather than concentrating on the response of individual nuclei, we compare the scaling function of *different nuclei* with mass number $A \geq 4$, and study the degree to which these scaling functions are the *same*. Our goal is to

explore whether the concept of superscaling, introduced in [7] within the relativistic Fermi gas (RFG) model, works for finite nuclei. For these studies we concentrate on the main part of the quasi-elastic peak where superscaling could, if anyplace, be hoped to work.

II. FORMALISM

Discussions of scaling at intermediate energies assume that inclusive electron scattering in the quasi-elastic regime has as a dominant process the impulsive one-body knockout of nucleons together with contributions from two-body meson exchange current (MEC) and meson production processes that may play a role when the normally dominant process is suppressed. In addition to the electron scattering angle θ_e, one has two variables to characterize the cross section, typically chosen to be (q, ω). Of course, any function $z = z(q, \omega)$ of q and ω may be used together with q to characterize the cross section. For over two decades it has been traditional to use the so-called y-scaling variable for $z(q, \omega)$ (for a review of the history of the subject, the basic formalism and reference to many of the theoretical studies undertaken see [6]). Upon dividing the inclusive electron scattering cross section by the single-nucleon electromagnetic cross section together with the Jacobian required in changing variables to y one obtains a derived function $F(q, y)$. Scaling means that at high enough values of momentum transfer this function becomes a function only of y, that is, independent of q:

$$F(q, y) \stackrel{q\to\infty}{\longrightarrow} F(y) \equiv F(\infty, y). \qquad (1)$$

Indeed, it has been found that in the $y < 0$ region (the region of the low-ω side of the quasi-elastic peak) for momentum transfers of roughly 0.5 GeV/c or larger y-scaling is very well obeyed. For more detailed discussion of this approach and for a summary of the experimental situation, see [6].

As an alternative to this standard plane-wave impulse approximation approach scaling can be examined from a different point of view using as a starting point the RFG model [7,8]. This model has the appeal of simplicity while maintaining important aspects in the problem such as Lorentz covariance and gauge invariance. Naturally it ignores potentially important effects such as those stemming from strong final-state interactions or two-body MEC and employs an overly simplified initial-state spectral function; nevertheless, such ingredients can be added to the basic model and appear not to invalidate it as a basic starting point for analyses of scaling.

Below we summarize the essential RFG developments and refer the reader to [8,9] for more detail, including the relationships between the RFG formalism and the usual y-scaling analysis. As seen in the work cited, a dimensionless scaling variable ψ naturally emerges:

$$\psi = \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau}{\sqrt{(1+\lambda)\tau + \kappa\sqrt{\tau(1+\tau)}}} \qquad (2)$$

where $\xi_F = \sqrt{1+\eta_F^2} - 1$ and $\eta_F = k_F/m_N$ are the dimensionless Fermi kinetic energy and momentum, respectively. Here we employ dimensionless variables $\kappa \equiv q/2m_N$, $\lambda \equiv \omega/2m_N$ and $\tau \equiv \kappa^2 - \lambda^2 > 0$. Additionally, to allow for the fact that nucleons are knocked out of all

shells in the nucleus we follow the spirit of [10] and introduce an "average removal energy" by shifting ω to $\omega' \equiv \omega - E_{rem}$ and hence defining a derived variable ψ' by substituting $\lambda' \equiv \omega'/2m_N$ for λ and $\tau' \equiv \kappa^2 - \lambda'^2$ for τ in Eq. (2). In fact, in the present work we simply adjust E_{rem} to fit the systematic trend seen in the data, rather than employing the specific shift discussed in [10]; however, since typically ω is large for the kinematics considered below, while the chosen values of E_{rem} are small, ranging from about 15 to 25 MeV in our analysis, the actual values chosen are not critical.

Weighting the squares of the single-nucleon form factors with the appropriate proton and neutron numbers, Z and N,

$$\begin{aligned}\widetilde{G}_E^2(\tau) &\equiv ZG_{Ep}^2 + NG_{En}^2 \\ \widetilde{G}_M^2(\tau) &\equiv ZG_{Mp}^2 + NG_{Mn}^2 \end{aligned} \tag{3}$$

within the RFG model it is then natural to define a function

$$F(\kappa, \psi') \cong \frac{k_F\, d^2\sigma/d\Omega_e\, d\omega}{\sigma_M[v_L(\kappa/2\tau)\widetilde{G}_E^2 + v_T(\tau/\kappa)\widetilde{G}_M^2]} \tag{4}$$

where terms of order η_F^2 have been ignored here since η_F is typically small, growing from 0.06 for deuterium to about 0.3 for the heaviest nuclei (full expressions are given in [8,9]). Here σ_M is the Mott cross section and $v_{L,T}$ the familiar Rosenbluth kinematical factors. In terms of the RFG variables scaling means that at high q (large κ) one finds that $F(\kappa, \psi')$ becomes only a function of ψ' – indeed, if E_{rem} is set to zero, then the RFG scales (in ψ) exactly, as it must, since this provided the original definition of ψ [7].

While the RFG approach discussed here and the usual y-scaling analysis have differences in detail, they are rather closely related under "typical" circumstances, namely, for momentum transfers that are relatively large (say $q > 0.5$ GeV/c), for energy transfers that are not too close to threshold (where both approaches are bound to fail) and for nuclei that are not too light (since the RFG approach cannot account for daughter nucleus recoil effects). Roughly speaking the ψ' variable can be regarded as an alternative to y/k_F and $F(\kappa, \psi')$ can be equated with $k_F \times F(q, y)$. One finds that scaling in ψ' and y is very similar (this is discussed in more detail in a longer accompanying paper [9]).

In [7] the idea of "superscaling" was introduced, but until now not tested with existing high-quality data on quasi-elastic scattering. The observation to be drawn from the RFG developments summarized above is that, within the context of that model, plotting $F(\kappa, \psi)$ versus ψ leads to a universal curve, namely, not only universal in that no κ-dependence remains, but also that the result *is independent of the choice of nucleus* modulo minor corrections of order η_F^2. This suggests attempting the same type of representation of the experimental data to investigate the idea of superscaling, which we proceed to do in the following section.

III. RESULTS

We next use the inclusive electron-nucleus scattering data in order to test the idea of superscaling. We also try to disentangle better the various reasons that underlie the well-known fact that at large electron energy loss nonscaling is observed. For these studies

we basically concentrate on nuclei with $A \geq 4$, as the lightest nuclei are known to have momentum distributions that are very far from the "universal" one which is at the basis of the superscaling idea.

Data on inclusive electron-nucleus scattering for a series of nuclei $A \geq 4$ are available in the region of low momentum transfers $q \sim 0.5$ GeV/c [11] – [24], while data extending to much higher q are available from other experiments [25] – [28]. Not all of these data can be used, however, as some of them have not been corrected for radiative and Coulomb distortion effects, are known to have problems such as "snout scattering" or the inclusion of false signals from π^-'s in the electron spectrometer, or are only available in the form of figures. Part of the data is at very low momentum transfer, which we do not consider as there scaling is known to break down due to large final-state interactions and Pauli blocking effects.

In a first step, we have taken the data that meet our criteria for the nuclei $A = 12 \ldots 208$ and have analyzed them in terms of scaling in the variable ψ'. As ψ' is defined in terms of the Fermi momentum, appropriate values of k_F had to be selected. We use 220, 230, 235 and 240 MeV/c for C, Al, Fe, Au, with intermediate values for the intermediate nuclei.

Figure 1 shows the scaling function $F(\psi')$ for all kinematics (energies, angles, momentum transfers) suitable for the present study and all A available. We clearly observe a scaling behavior for values of $\psi' < 0$: while the cross sections at a given ψ' vary over more than three orders of magnitude, the values of $F(\psi')$ are essentially universal. For $\psi' > 0$, on the other hand, the scaling property is badly violated, and this is to be expected, as there processes other than quasi-elastic scattering – meson exchange currents, Δ-excitation, deep inelastic scattering – contribute to the cross section. The scaling as discussed in this paper applies only to processes having the kinematics of electron-nucleon quasi-free scattering.

In order to separate some of the effects leading to less-than-perfect scaling at negative ψ', in Fig. 2 we show the function $F(\psi')$ for the series of nuclei $A = 12 \ldots 197$, but for fixed kinematics (electron energy 3.6 GeV, scattering angle 16°, and hence nearly constant q). The quality of the scaling in the region $\psi' < 0$ is quite amazing. This shows that, insofar as the removal of the A-dependence goes, the superscaling works extremely well. The deviations from scaling observed in Fig. 1 are *not* from an A-dependence.

A part of the A-dependent increase of $F(\psi')$ at positive ψ' results from the increase of k_F with A. This amounts to an increase of the width of the quasi-elastic and Δ peaks, and a correspondingly increased overlap with non–quasi-free scattering processes (Δ-excitation, π-production, ...). At the same time, the increasing average density of the heavier nuclei may also lead to an increase in contributions of two-body MEC processes which are presumably strongly density-dependent (i.e., do not scale with k_F in the same way the one-body knockout processes do; see [29] for indications of this type of behavior).

Figure 3 shows the data for $A = 4$, 12, 27, 56, 197 on a logarithmic scale for the kinematics of Fig. 2. This figure demonstrates that the scaling property extends to large negative values of ψ', values which correspond to large momenta of the initial nucleon. A priori, this feature is not predicted within the RFG model used to motivate the choice of ψ'. It can be understood, however, from the theoretical results for the momentum distribution of nuclear matter as a function of the nuclear matter density. For different nuclear matter densities, the tail of the momentum distribution $n(k)$ at $k > k_F$ (corresponding to $\psi' < -1$) is a near-universal function of k/k_F [30]. For finite nuclei and large momenta we can employ the Local

Density Approximation (LDA), as at large k we are dealing with short-range properties of the nuclear wave function [31]. Within LDA, the nuclear momentum distribution (spectral function) then is a weighted average over the corresponding nuclear matter distributions. This means that the large-momentum tail of the nuclear momentum distribution also scales with k_F, a dependence that is removed when using ψ'.

In order to emphasize the quality of this superscaling in the tail, in Fig. 3 we have also included the data on ^{4}He, taken under the same kinematical conditions [25]. While at ψ'=0 $F(\psi')$ for ^{4}He is about 10% higher than for heavier nuclei as a consequence of the sharper peak of the momentum distribution $n(k)$ at $k \sim 0$, the scaling function for ^{4}He agrees perfectly with the one for heavier nuclei for $\psi' < -0.2$.

A similar quality of superscaling is found when analyzing the data at the other kinematics at both higher and lower q where cross sections for a large range of A are available. As an example, Fig. 4 shows the scaling function for the data at 3.6 GeV and 25° [25]

The quality of scaling in terms of the variable ψ' is quite similar to the one found when using the standard variable y. We have systematically used ψ' defined above as this variable is more directly connected to the RFG model that motivated the study of superscaling.

IV. CONCLUSIONS

We have analyzed data on electron-nucleus quasi-elastic scattering for nuclei with mass numbers $A = 4 - 208$ which cover a large range in q, ω. We find that, upon use of the scaling variable ψ' which allows one to remove the "trivial" dependence on the Fermi momentum, the data on the low-ω side of the quasi-elastic peak ($\psi' < 0$) show superscaling, *i.e.* the scaling functions $F(\psi')$ for the different nuclear mass numbers A coincide. The A-independence of the superscaling function actually is much better realized than the q-independence of the normal scaling function, which is violated due to processes such as meson exchange currents and/or excitation of internal degrees of freedom of the nucleon. The realization of superscaling shows that different nuclei, in the integral sense tested via inclusive scattering [6], have a more or less universal momentum distribution once the obvious dependence on the Fermi momentum k_F is removed.

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FIGURES

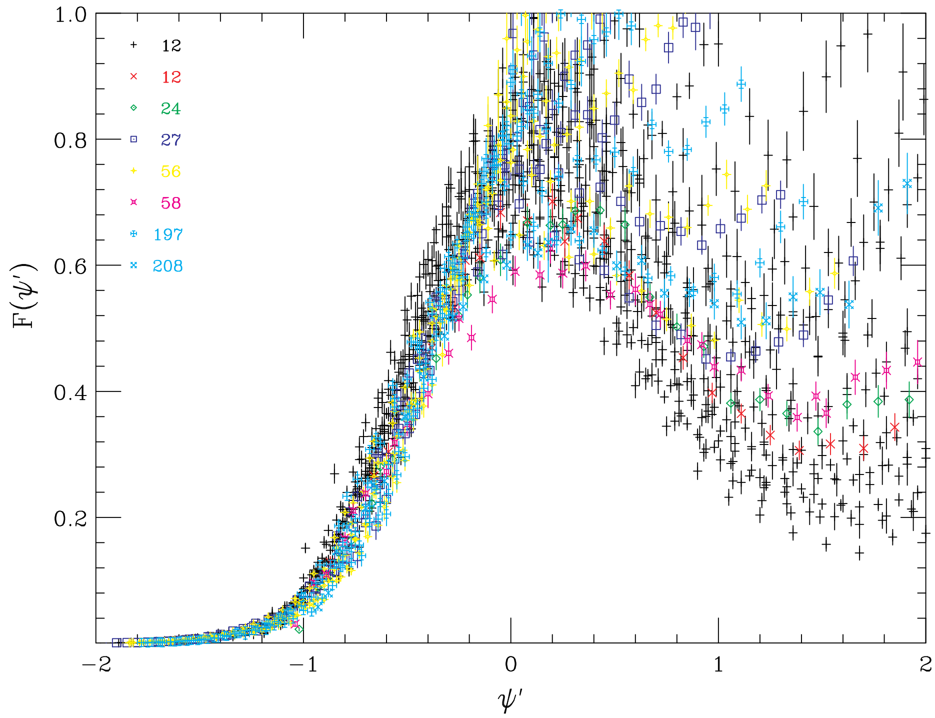

FIG. 1. Scaling function $F(\psi')$ as function of ψ' for all nuclei $A \geq 12$ and all kinematics. The values of A corresponding to different symbols is shown in the insert.

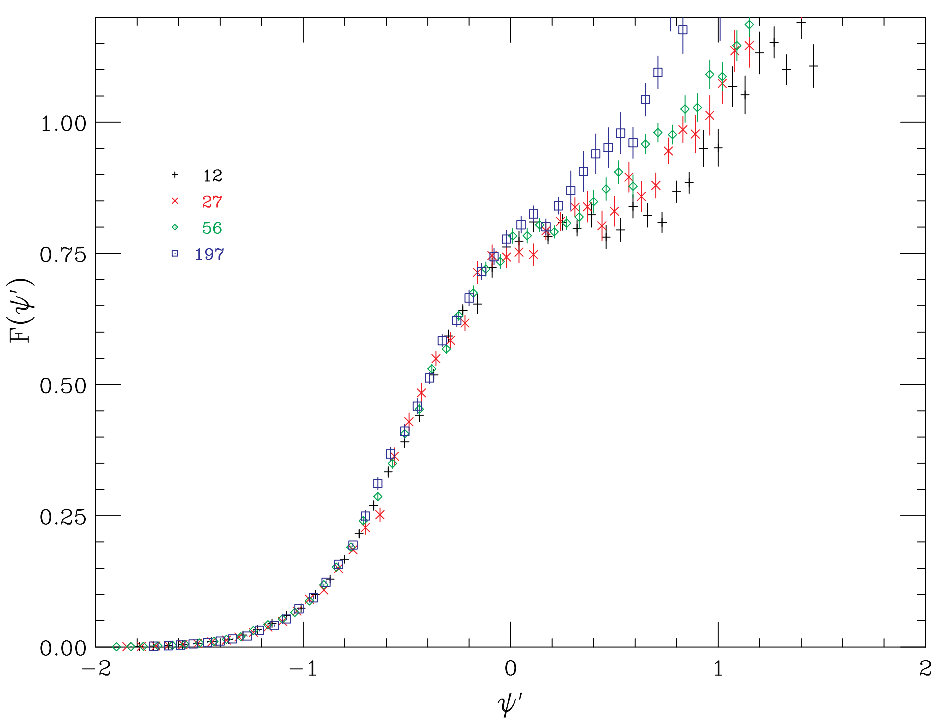

FIG. 2. Scaling function for C, Al, Fe, Au and fixed kinematics [25]. The correspondence of symbol and mass number of the nucleus is shown in the insert.

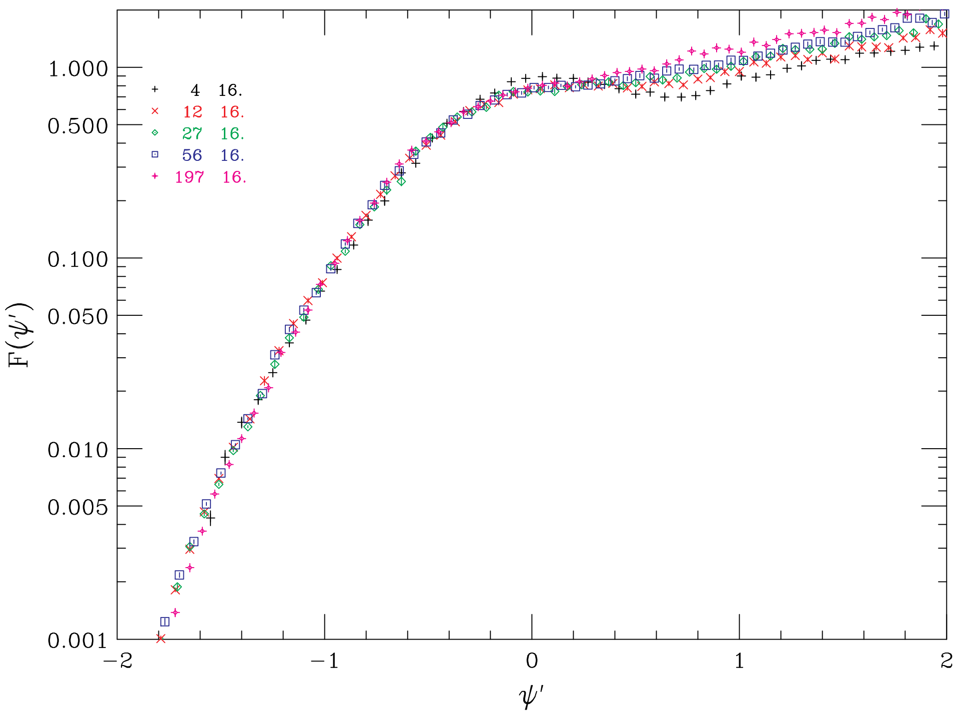

FIG. 3. Scaling function for nuclei $A = 4 - 197$ and fixed kinematics on logarithmic scale.

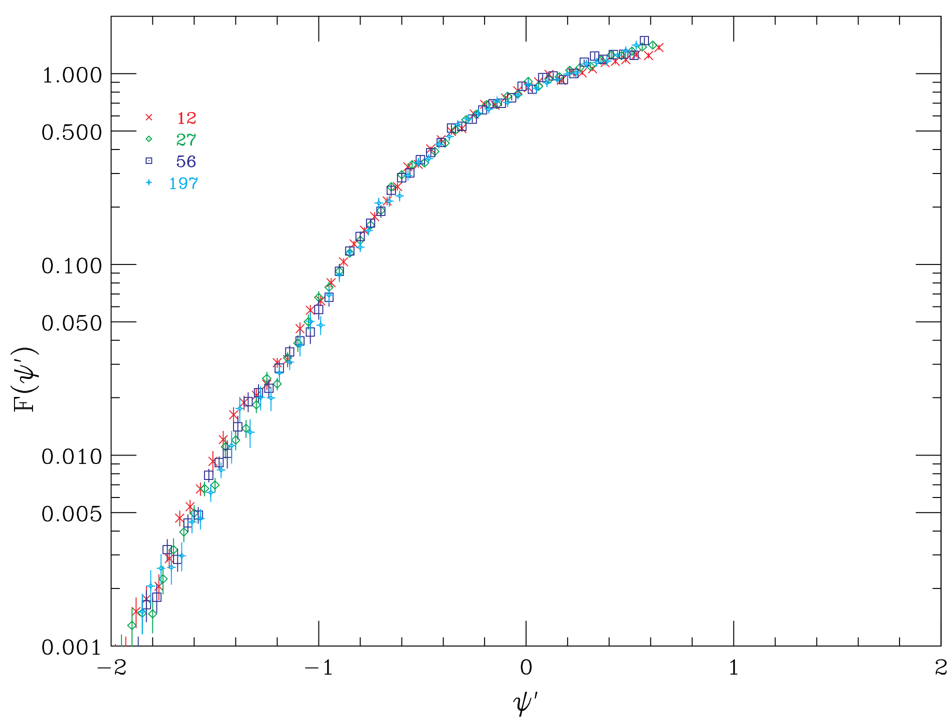

FIG. 4. Scaling function for nuclei $A = 4 - 197$ at higher momentum transfers (3.6 GeV, 25°).

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